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Experimental and analytical investigation of the gas filtration combustion inclination instability

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Abstract

Investigation on state of the art of thermal-hydrodynamic instability of gas filtration combustion is presented. Development of the model of flow competition which takes into consideration thermal and filtration reorganization of the system as reaction to the front perturbation is proposed. Existence of characteristic and critical front inclination is demonstrated experimentally. It is shown that in the condition of experiments, characteristic inclination can be estimated by the system length and wave dimensionless velocity $\Delta X_{\mathrm{ch}} \sim L \cdot u$. The inclination amplitude growth velocity is found to be proportional to wave front velocity $\Delta\dot{X} \approx 0.4 \cdot u_{\mathrm{w}}$. © 2001 Elsevier Science Ltd. All rights reserved.

1. Introduction

Gas combustion in a porous media or filtration combustion (FC) is under intensive investigations now due to a large number of technical application such as VOCs oxidation, lean combustible mixtures burning under superadiabatic condition, catalyst treatment by thermal wave, etc. [1,2].

The term "filtration combustion" in a wide meaning covers exothermic processes taking place in multi-phase systems. It includes combustion of solid fuels during oxidizer filtration and multi-component chemical transformations during self-propagated high-temperature synthesis, liquid fuel combustion in inert porous body, etc. The present work considers gas filtration combustion in inert porous medium, though some obtained regularities can be applied to other cases of FC too.

FC wave front can be unstable, i.e. not preserving its startup geometry, which is a basic obstacle to widely use the FC devices in industry. From the physical viewpoint, the instability is a non-steady growth of FC front perturbation, which leads to combustion extinguishing or to emergence of new (stable) front structure. The inclination and hot-spot instability were distinguished in experiments [3,4]. Inclination instability manifests itself in gradual loss of initial orientation of the front (perpendicular to the filtration vector) that leads to front decay or fragmentation, if instability continues to develop. Hot-spot instability is front perturbation that leads to separation (lagging from main front motion) of isolated hot sites of some spherical configuration.

There is no satisfactory solution of FC wave stability problem today, though this matter is discussed in a number of works [3–7]. When analyzing hydrodynamic instability in [4,5], an analogy with Landau theory of normal flame stability [8] is used.

Vainstein [5] solves heat and hydrodynamic instability problems independently. One-dimensional heat perturbations are considered for heat problem. A conclusion on absolute stability of FC front to heat perturbations in a wide range of parameters follows from [5], which is a rather evident fact when considering analogy with the problem of one-dimensional normal flame stability. Analysis of hydrodynamic instability in [5] considers 2D problem. As shown, provided that filtration coefficients do not differ strongly on both sides of combustion front, the flame is hydrodynamically unstable.

Minaev et al. [4] studied FC stability problem experimentally and theoretically. Inclination instability of

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Nomenclature

c	heat capacity
D	burner tube diameter of perturbation transverse size
d_0	diameter of particle of porous bed
f''	front local curvature
G	gas mass flow rate
h	perturbation amplitude (cavity mean depth)
H	gas-dynamic clog (hot zone) width in the porous body
k	filtration permeability
l_{th}	length of thermal relaxation
l_{hd}	hydrodynamic relaxation length
L	total length of porous body
P_0	pressure at the outlet of burner
ΔP	pressure drop in burner
ΔP_{cold}	pressure drop in the cold (not operating) burner
p_1	mean pressure in the coordinate of the front
$\tilde{q}$	specific heat flux to the preheating zone of unperturbed front
Q_λ	heat flux to the preheating zone mean over perturbation cross-section
t_c	perturbation compensation time
T_{max}	maximum temperature in the filtration combustion wave front
T_0	initial or ambient temperature
$u_w, u_{w,0}$	flame front velocity and unperturbed flame front velocity
u_t	thermal wave velocity in porous media
$u = u_w/u_t$	dimensionless wave front velocity
u_g	gas filtration velocity
x	coordinate along the axis of the burner

Greek symbols

β	coefficient of Newtonian heat losses of the burner, $W/m^3/K$
γ	positive constant in (1)
λ	thermal conductivity of porous bed
Δ	variation or difference
δ_P, δ_T	initial perturbation amplitude parameters
ΔX	inclination amplitude (coordinates difference between "head" and "tail" of combustion front)
$\Delta\dot{X}$	time derivative of inclination amplitude
μ	gas viscosity
ρ	density
σ	Stephan–Boltzmann constant
τ_{fr}	frontal thermal relaxation time
$\phi = \tilde{q}_\lambda/\tilde{q}_u$	dimensionless factor

Subscripts

cr	critical value
ch	characteristic value
g	gas
s	solid
λ	having conductive nature
u	having convective nature
min	minimum value

the front was observed in a quartz tube of 40 mm diameter at downstream FC regimes. Unfortunately, the work does not contain any information on the peculiarities of inclination instability dynamics, front break conditions, etc. The problem was considered analytically for long-wave perturbations (considerably exceeding front width). A case of short-wave perturbations was also considered by means of parameterization of the front local velocity via front curvature f'' (Markstein method) $u_w = u_{w,0}(1 + \gamma f'')$ (γ – positive constant). As a result of the above assumption and the fact that the model indirectly (via boundary conditions) considers peculiarities of the thermal problem, the authors obtained a formula for perturbation critical size, practical use of which is straitened because of the undefined γ.

Report [7] solves the problem of thermal FC front instability for small 2D perturbations of the front in approximation of instantaneous reaction. Reaction rate variation with temperature was considered by Frank–Kamenetsky expansion of Arrhenius function. Perturbation amplification factor is determined and critical perturbation wavelength is defined. One of the results is higher growth rate for longer wavelength perturbations. Results [7] qualitatively agree to the conclusions made by Minaev et al. However, their detailed interpretation needs discretion, since, first, the results are related to only small perturbations and, second, a special type of perturbations is considered at which macroscopic area of perturbation gains (or looses) energy at unchanged front velocity.

One of the possible approaches to study the stability of thermodynamic systems to final amplitude perturbations is an energy method, first offered by Orr [9] to study convective instability (Bernar's problem). At present there are no works dedicated to FC system analysis based on the energy method.

Thus, there is no satisfactory theory of FC wave stability that, by our opinion, must, first, consider non-stationary problem for perturbations, second, regard multi-dimension of the perturbations, third, include interaction of hydrodynamic (perturbation of filtration field) and thermal factors and last, consider not only local front characteristics, but system in hole. On the other hand, even rather huge 2 and 3D numerical models of FC can hardly give reliable results on instability for real systems because of principle non-uniformity of porous media, significant role of turbulence [10], peculiarities of chemical kinetics, etc.

A certain progress in solving FC instability problems has been made recently [11–13]. New methods to analyze the stability, namely, the method of flow competition (MFC) to analyze dynamic behavior of perturbations [11], filtration problem small perturbation (FSPA) analysis [12] as well as method that unites the above approaches [13] were proposed. The said methods let us to obtain estimates for instability criterion and describe some qualitative peculiarities of wave front dynamics.

In the present work short description of the named methods is presented and development is given, which includes analysis of system reaction on front inclination. Dynamics of inclination instability is studied experimentally. The theoretical predictions of existence of characteristic and critical front inclinations are verified and formulas specifying those parameters are revised on the basis of experimental data.

2. Theory of thermal–hydrodynamic instability of filtration combustion front

The method of flow competition (MFC) [11] considers the heat balance of combustion front preheating zone in the cross-section of perturbation (cavity), Fig. 1. Physically, perturbation amplification is caused by filtration field redistribution near the flame front in such a way that the total gas flow rate in the perturbation cross-section grows with the perturbation amplitude increase. At the same time, competing process of conductive and radiative heat transfer into the preheating zone compensates perturbation growth.

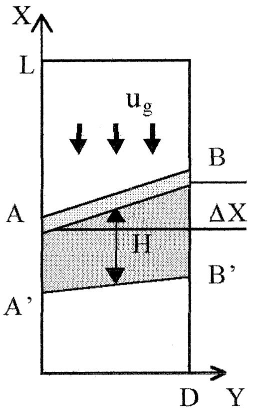

Fig. 1. Filtration combustion front inclination scheme. H – hot zone width, AB – wave front, A′B′ – wave rear. Half-shaded area – front preheating zone.

Assuming the temperature of the front is constant independent of perturbation and using simple models for evaluation of the conductive Q_λ and convective Q_u heat fluxes as a function of the perturbation amplitude h [11,13]

$$Q_\lambda = \frac{\pi D^2}{4}\left(1 + 4(h/D)^2\right)\tilde{q}_\lambda, \tag{1}$$

$$Q_u = \frac{\pi D^2}{4}\left(1 + \left(\frac{h}{2} + \frac{2}{3}\frac{h^3}{D^2}\right)\Big/(H-h)\right)\tilde{q}_u \tag{2}$$

one can analyze perturbation dynamics. Here $\tilde{q}_\lambda, \tilde{q}_u$ are specific conductive and convective fluxes for unperturbed front, h the perturbation cavity mean depth , D its transverse size, and H is the width of gas-dynamic clog (hot zone) in the porous body.

According to MFC a small perturbation can grow if convective flux accretion caused by the perturbation exceeds the competing growth of the conductive flux, $\Delta Q_u > \Delta Q_\lambda$, or regarding (1), (2)

$$\frac{4h}{D}\tilde{q}_\lambda < \frac{D}{2(H-h)}\tilde{q}_u + \frac{2}{3}\frac{h^2}{(H-h)D}\tilde{q}_u. \tag{3}$$

Analysis of quadratic relative to h inequality (3) shows that small in amplitude perturbations $h \ll D, h \ll H$ always grow. Depending on discriminant of Eq. (3), perturbation growth can either stop at a certain characteristic amplitude $h = h_{ch}$

$$h_{ch} = \frac{3H\phi}{1+6\phi}\left(1 - \sqrt{1 - \frac{D^2}{H^2}\frac{1+6\phi}{12\phi^2}}\right) \tag{4}$$

or continue to grow up to front break. Here notation $\phi = \tilde{q}_\lambda/\tilde{q}_u$ is used, In amplitude interval $h_{ch} < h < h_{cr}$ perturbations cannot grow and fall to h_{ch}. If due to considerable fluctuation or external effect, perturbation amplitude exceeds the critical value $h > h_{cr}$, where

$$h_{cr} = \frac{3H\phi}{1+6\phi}\left(1 + \sqrt{1 - \frac{D^2}{H^2}\frac{(1+6\phi)}{12\phi^2}}\right), \tag{5}$$

perturbations will grow up to the front break.

Condition of absolute front instability for the considered geometrical class of perturbations follows from discriminant negatively of (3)

$$D/H > (D/H)_{cr} = 2\phi\sqrt{3/(1+6\phi)} \approx \sqrt{2\phi}. \tag{6}$$

The width of gas-dynamic clog H is the most important macroscopic parameter of the system that can be expressed by easily measurable values – total length of porous body L, pressure drop in the cold ΔP_{cold} and hot ΔP (operating) systems at same gas flow rate and maximal temperature in the wave front T_{max} [13].

$$H = L\frac{\Delta P/\Delta P_{\mathrm{cold}} - 1}{(T_{\max}/T_0)^{3/2} - 1}. \qquad (7)$$

In the work [12] filtration small perturbation (FSP) amplification analysis was performed by using Darsi–Leibenzon [14] filtration equation

$$\frac{\partial(P^2)}{\partial t} = (k/\mu)P_0\Delta(P^2). \qquad (8)$$

Here k is filtration permeability, μ the viscosity, and P, P_0 are the local pressure and pressure at the outlet, respectively. In [13] critical transverse size of perturbation, condition of absolute instability and other characteristics are determined. The value of the results obtained with FSPA is limited due to, first, assumption of perturbation smallness and, second, to absence of system macroscopic characteristics in the problem statement and analysis. If we formally apply the obtained results to arbitrary amplitudes of perturbation, we come to qualitatively the same results as MFC method arrives at. In [13] a possibility to unite both MFC and FSPA models in a way to include both local and macroscopic system characteristics is discussed.

The models described above predict important property of the FC wave: existence of characteristic and critical perturbation amplitudes and absolute instability condition. The corresponding correlation for characteristic inclination and critical hot zone width are presented in Table 1. Note that equations of Table 1 may be formulated by using equity $q_\lambda/q_u = (1-u)$. According to Table 1 MFC model predicts small characteristics inclination $\Delta X_{ch}/D < 1$ (at least for slowly propagating waves $u \leqslant 0.1$) and its inverse dependence on H. Instability criterion is expressed as hot zone width smallness condition according to all models. The critical hot zone width is close to unity $H_{cr}/D \sim 1$ in the case of $u \leqslant 0.1$.

The experiments described below in the paper show some contradictions with the results of the above models: (1) inclination grows at hot zone width exceeding its critical value, (2) front stabilization inclination ($\Delta X/D = 1.5$–2), noticeably exceeds MFC estimates of characteristic inclination, (3) inclination directly correlates with dimensionless wave velocity and independent of hot zone width.

The first may be attributed to 2D and wall effects important in practical systems. Really, the effective porosity of near wall layers is up to 50% higher [15] and temperature is noticeably lower than average in cross-section. This leads to a situation in which hydrodynamic clog width near the walls is less than average by cross-section and filtration asymmetry starts up near the walls. This notion leads to conclusion that in real 2D and 3D systems, instability condition (Table 1) should be related for locally minimal hot zone width, not average by cross-section. Explanation of the second and third points should be sought in thermal and hydrodynamic reorganization of the system that is not taken into account in the above-described models and that can be treated as some factor of negative feedback to perturbation growth. Let us term the instability conditions implied by MFC model a primary instability condition, which can be treated as necessary (but not sufficient) condition for front break, taking into mind that the MFC model does not consider system thermal and hydrodynamic reorganization.

It is impossible to strictly analyze 2D evolution of the corresponding temperature and filtration fields, although some physical assessments could be done.

The main physical processes that influence the thermal reorganization of the system could be schematically described as follows: (1) relative movement of front sections in the process of inclination causes local temperature differentiation of these sections in accordance with local heat balance (similar to correlation between the FC wave velocity and $T_{\max}$); (2) in the case of non-uniform filtration field, the enthalpy flux to some parts of cross-section will be higher than to others which will lead to gradual temperature growth in the corresponding part of the front (a point area in the Fig. 1); (3) filtration flux redistribution occurring in the front also takes place at some distance after the front due to filtration flux continuity even in the case of homogeneity of the rest of porous media. The corresponding variation of the filtration field in the hot area increases heat transfer along x-axis, elongates high temperature zone

Table 1
Characteristic perturbation inclination and instability critical value for H as defined by the method of flow competition (MFC), filtration small perturbation (FSP) analysis and united models

	MFC		FSP		United model	
$\frac{\Delta X_{ch}}{D}$	$\frac{1}{4}\frac{D}{H}\frac{q_u}{q_\lambda}$	(9)	$\frac{4}{\pi}\frac{q_u}{q_\lambda}$	(10)	$\left[\frac{4}{\pi}+\frac{4}{\pi^2}\frac{D}{H}\right]\frac{q_u}{q_\lambda}$	(11)
$\frac{H_{cr}}{D}$	$0.7\sqrt{\frac{q_u}{q_\lambda}}$	(12)	$\frac{2}{\pi}\frac{q_u}{q_\lambda}$	(13)	$\frac{1}{\pi}\frac{q_u}{q_\lambda}\left[1+\sqrt{1+\frac{2q_\lambda}{q_u}}\right]$	(14)

and thus suppresses filtration flux increase in the given cross-section; (4) when front inclination considerably grows heat exchange interface increases. This result in mean hot zone temperature decrease and other effects reducing homogeneity and stability of front, particularly direct flame quenching.

Separation of the said mechanisms is most conditional and related to the fact that they can be analyzed within the given separation. It is important to note that all the mentioned physical processes are transient. The characteristic time of the temperature field reorganization is determined by the thermal inertia of the solid. The first of the mentioned mechanisms asymptotically disappears and can be neglected if inclination stabilizes. Consider the second. The temperature increase due to additional enthalpy flow is proportional in the first approximation to the gas flow rate increase $\Delta T/T \sim \Delta u_g/u_g$. If one will formally multiply right-hand side of Eq. (1) by the appropriate $\Delta u_g/u_g$ correlation and apply MFC principles, one will conclude that conductive flux increase always compensates convective flux fluctuation ($\delta Q_\lambda > \delta Q_u$) and front inclination is impossible. This contradiction is explained by thermal inertia time delay of the solid heating up. This process has a nature of relaxation to steady state $\Delta T(x,y,t) = \Delta T(x,y)(1-\exp(-t/\tau))$, where characteristic time can be estimated via some system linear scale and thermal wave velocity. Consider the third mechanism. If the filtration velocity perturbation takes place in vicinity of the front, it will continue in the hot zone and relax at some characteristic length scale. The basic functional dependence (see Appendix A) has the functional form $\exp(-\pi x/D)$ and typical relaxation length is some part of system transverse size $\sim D/\pi$. Porous body temperature relaxation length along the filtration pipeline determined by one-dimensional and one-temperature model [16] is proportional to filtration velocity

$$l_{th} = u_g c\rho(1-u)/\beta. \tag{15}$$

Thus, the width of hot zone will tend to increase in the cross-section area, where filtration velocity increases and consequently will tend to compensate the initial filtration inhomogeneity.

The above analysis demonstrates that initial perturbation of the front geometry causes system thermal reorganization which tends to compensate filtration redistribution and front asymmetry. In the case of non-inertial solid (zero heat capacity) quick and complete compensation will secure stability of the front geometry even if primary instability condition (Table 1) takes place.

The instability of the practical FC systems is explained by final time of system thermal reorganization. Time scale parameter having the meaning of thermal inertia of the system should be introduced to the instability model. At the times bigger than thermal inertia times, perturbation stabilizes or decreases. Simplest estimate for thermal relaxation time is $\tau_{fr} = H/u_t$ and for perturbation growth time is $\tau_{pert} = H/u_w$. Given estimate secures inequity $\tau_{fr} \leqslant \tau_{per}$ as far as $|u_t| \geqslant |u_w|$ meaning considerable compensation of perturbation growth. The steady FC wave is characterized by $\tau_{per} \to \infty$ and consequently perturbation growth is prohibited. This effect is responsible for stability of steady-state FC waves reported in experiments.

The simplest model based on the above concept can be formulated as follows. If primary instability condition is not fulfilled, perturbation amplitude remains on the characteristic level defined by Eqs. (9)–(11) of Table 1. Otherwise inclination grows with the velocity proportional to u_w. Note that Eqs. (9)–(11) use average hot zone width H, (12)–(14) relate to locally minimal H_{min} value. Thermal reorganization of the system goes on in rival with characteristic velocity u_t. The time which is necessary for perturbation compensation is a complex function of system parameters and cannot be obtained without multi-dimensional problem solution. Nevertheless, some upper estimate is given by the porous body total length and thermal wave velocity

$$t_c \sim L/u_t. \tag{16}$$

Some lower estimate is given by the critical width of hydrodynamic clog, as far as H_{cr} is the length controlling stability of the FC front

$$t_c \sim H_{cr}/u_t. \tag{17}$$

Inclination stabilization level can be estimated by using (16)

$$\Delta X_{st}/D = \frac{u_w t_c}{D} \sim \frac{L}{D} u \tag{18}$$

or (17) correspondingly

$$\Delta X_{st}/D = \frac{u_w t_c}{D} \sim \frac{H_{cr}}{D} u. \tag{19}$$

As shown below estimate (18) gives reasonable approximation to experimental data.

Front cannot stabilize at arbitrarily high angle. In some cases heat loss implies limit to the maximum possible inclination. In other cases it is determined by the system dimensions. In fact, inclination development assumes relatively narrow hot zone width (primary instability condition). Inclination amplitude ΔX cannot exceed considerably the H value with the front integrity conserved. This gives estimate for maximum possible inclination as

$$\Delta X_{max}/D \sim H/D. \tag{20}$$

More accurately this parameter can be defined experimentally.

3. Experiment and discussion

To verify the model, inclination instability of FC front was studied experimentally in cylindrical tube at different flow rates and mixture equivalence ratios. We observed FC wave inclination growth rate, characteristic inclination (in case of front stabilization), critical conditions for flame front break.

Burner was a quartz tube with inner diameter of 41 mm, wall thickness of 3 mm and length of 650 mm. Methane and air mixture was used as combustible. Gas flow rate and composition were controlled by rotameters with an accuracy of 10%. The range of air flow rate was 1500–3000 l/h and that of methane 70–280 l/h. Within the experimental sets, the flow rates were varied while fuel-to-air ratio was maintained at a constant. The pressure drop at the burner (which is necessary for H evaluation) was measured by U-gauge and varied between 0.01 and 0.1 atm. Four series of tests were performed with a bed of Al_2O_3 balls with an average diameter $d_0 = 3.5$ mm, porosity – 0.4; Al_2O_3 heat capacity – 1250 J/kg/K, density – 1.7 kg/m^3. Fifth set of tests was done with hollow Al_2O_3 balls $d_0 = 3.5$ mm, effective density – 0.86 kg/m^3 as a bed filing. Ignition was made by spark plug placed into the bed. To improve ignition, large ceramic particles $d_0 = 7$ mm were placed in the area of electrodes (mixture failed to be ignited in the small balls). Ignition was performed at nearly stoichiometric mixture and after visible hot zone was formed over complete cross-section, operation parameters were switched on. The balls of the inert bed being heated FC wave was formed and propagated downstream the system. Filtration flow direction is taken for positive direction of the wave propagation. After front reached the last third of the burner, measurements were stopped.

Front "head" and front "tail" position were measured with optical cathetometer at fixed time intervals as an average result of several sequential measurements. Estimated accuracy of measurement being 15%. The front velocity u_w was defined as the mean of the head and tail velocities. Dimensionless inclination of the front was defined as difference between the head and tail positions related to the tube diameter $\Delta X/D$. Average gas filtration velocity u_g was calculated regarding volumetric flow rate, tube cross-section and porosity. Heat wave velocity was calculated according to definition

$$u_t = \left\langle \frac{c_g \rho_g}{c_s \rho_s} \right\rangle \frac{m}{1-m} u_g,$$

where the brackets mean averaging by temperature interval $T_0 \ldots T_{max}$, accepted porosity $m = 0.4$. The average width of gas-dynamic clog H was defined by formulae (8). The average by the cross-section combustion front temperature was evaluated as the mean one between adiabatic temperature of the FC wave $T_{ad}/(1-u)$ (with respect to wave propagation [13]) and temperature of the solid near the external boundary, that was measured by pyrometer. This assessment overestimates mean temperature of porous body, although discrepancy lies within 10%.

Fig. 2 (left photograph) shows filtration combustion wave with stabilized inclination of the front. The width of the visible glowing zone is close to the calculated value (8) of gas-dynamic clog. The right photo in Fig. 2 shows wave front break.

After the ignition and hot zone width stabilization, the visible wave front was perpendicular to filtration vector and had natural perturbations with the amplitude having the order of the bed grain size d_0. The characteristic time of hot zone width stabilization was ~2–3 min. After this time T_{max} and ΔP were measured for H estimation and FC wave history was registered. The front moving, its inclination grew and subsequently could stabilize or result in front break. Dynamics of front inclination for four test series are presented in Fig. 4. The graphs in Fig. 4 demonstrate that in some tests FC front inclination reaches its saturation and evidences no tendency to grow. In the other tests inclination grows approximately linearly until front break. These results confirm the idea of characteristic inclination and more generally characteristic perturbation amplitude existence in the FC systems, which was first proposed in [12,13]. At the same time experiments show direct correlation between the measured characteristic inclination $\Delta X_{ch}/D$ and H/D ratio. Table 2 summarizes results of five series of experimental measurements. Primary experimental data: fuel to air volumetric ratio, gas filtration velocity at input conditions, FC wave velocity, stabilization inclination and FC hot zone width are presented in columns 2–7.

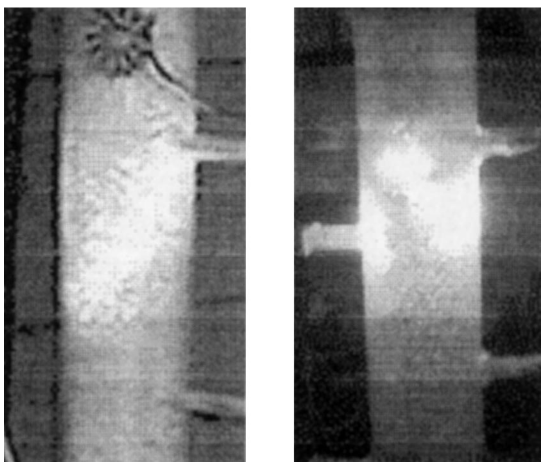

Fig. 2. Propagating filtration combustion wave with stabilized inclination (left) and broken front.

Table 2
Results of experimental measurement and theoretical estimation of characteristic inclination and FC dynamics parameters[a]

N series	F/A ratio	u_g, m/s	$U_w \times 10^4$, m/s	U	$\Delta X/D$	H/D	H/D, Eq. (14)	$\Delta X_{ch}/D$, Eq. (10)	$\Delta X_{ch}/D$, Eq. (11)	$\Delta X_{ch}/D$, Eq. (18)
1	2	3	4	5	6	7	8	9	10	11
1	1/14	0.99	0.65	0.12	1.2	1.44	0.96	1.45	1.77	1.46
	1/14	1.10	0.78	0.128	1.45	1.57	0.97	1.46	1.76	1.56
	1/14	1.18	0.9	0.135	1.9	1.78	0.98	1.47	1.74	1.65
	1/14	1.24	1.0	0.145	Break	2.1	0.99	1.49	1.71	1.77
2	1/15	1.06	0.9	0.153	1.75	1.67	0.99	1.5	1.79	1.86
	1/15	1.29	1.6	0.223	Break	2.16	1.06	1.64	1.88	2.72
	1/15	1.38	1.7	0.225	Break	2.15	1.07	1.64	1.89	2.74
3	1/16	0.94	1.0	0.19	1.75	1.25	1.03	1.57	1.97	2.3
	1/16	1.12	1.4	0.223	2.14	1.74	1.06	1.64	1.94	2.72
	1/16	1.17	1.5	0.23	2.2	2.1	1.07	1.65	1.9	2.8
4	1/17	0.99	1.3	0.235	1.75	1.68	1.08	1.66	1.98	2.86
	1/17	1.19	1.7	0.25	Break	2.25	1.1	1.7	1.94	3.05
	1/17	1.5	2.4	0.375	Break	2.3	1.27	2.0	2.3	4.5
5	1/15	0.97	1.5	0.147	1.4	1.9	0.94	1.49	1.65	1.79
	1/15	1.1	1.95	0.162	1.8	2.22	0.95	1.52	1.63	1.97
	1/15	1.33	3.3	0.225	2.2	2.35	0.99	1.64	1.7	2.74

[a] Burner: quartz tube with diameter $D = 41$ mm, length $L_{total} = 500$ mm, bed grain $d = 3.5$ mm. Air–methane mixture.

The experimental results may be described by using the models presented in Table 1. The measured values for H are higher than its critical values defined by Eqs. (12)–(14), see columns 7 and 8 in Table 1. This supposes inclination growth but only within the limits defined by (9)–(11). Characteristic inclination calculated according to(9) is much less than experimentally measured, although Eqs. (10) and (11) give reasonable (within accepted accuracy) agreement, see columns 9 and 10. Nevertheless, there are serious arguments against direct usage of models [11–13]. First, measured inclinations directly correlate with dimensionless wave velocity and hot zone width. Eqs. (9) and (11) do not suppose this correlation (see column 10), although (10) gives it via q_u/q_λ. Second, FSP and United model are based on small perturbation analysis and quantitative use of correlation (10), (11) is doubtful. Physical analysis performed in this paper is more adequate for description of the experiments by our opinion. Although measured H value is higher than instability critical limit, its local minimum value may be 30–50% lower as numerical estimates demonstrate. This means that primary instability conditions may realize for all experiments. Stabilization inclination calculated by (18) gives reasonable estimate and correct correlation with dimensionless wave velocity as presented in column 11 of Table 2. The front break up limit estimate (20) reasonably corresponds to experimental data too.

Important point of the experiments is in the following. The T_{max} is a unique characteristic of the system only in the beginning of the wave propagation. With the wave front inclination, maximum temperature difference in the wave head and tail was 100 K and more. At the same time mean temperature of the front decreases due to heat loss interface growth. After inclination development even the temperature of the head of the front may become lower than average front temperature in the beginning of the process, Fig. 3. To avoid double meaning of the models all measured and estimated parameters, characterizing stability of the FC system should be related to unperturbed geometry of the front.

In the fifth set of experiments made with hallow alumina particles, the inclination remains stabilized at some higher values of $\Delta X/D \approx 2.2$. The possible explanation is higher radiation conductivity of the hollow particles. (As reported in [17], photon freepath in hollow alumina particles bed is ~1.6 times higher than in the bed of solid particles.) This may result in better thermalization (temperature homogenization) of combustion wave and some wider stability limit. To clear up this question more experiments are necessary.

Fig. 4 shows that inclination amplitude grows approximately linearly until its saturation. We estimated linear speed of the amplitude growth $\partial\Delta X/\partial t = \Delta\dot{X}$ for the experiment runs and dimensionless $\Delta\dot{X}/u_w$ ratio, see Table 3. We included in Table 3 the data extracted from the photographs of the work [4], where experiments were performed with propane–air mixture. One can see that amplitude growth speed does not correlate with dynamic parameters within the experimental series. Dimensionless amplitude growth speed defined via FC wave velocity lies in interval from ~1/4 to ~ 1/2 with average

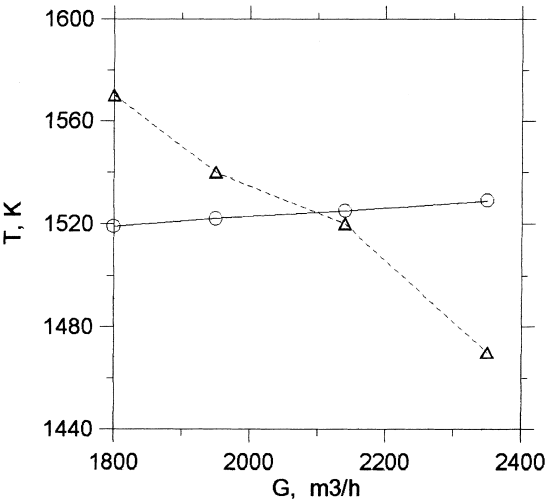

Fig. 3. Temperature of the porous media boundary layer for tests set 1, as measured for normal wave (3 min after ignition) – O, and in the "head" of inclined wave (30 min after ignition) – △.

value of ~0.4. More accurate determination of this parameter demands new experiments and simulation.

4. Conclusion

The main result of the investigation is the experimental confirmation of existence of the characteristic value of front inclination (inclination saturation). This result corresponds to FC instability understanding achieved in [11–13] and advanced in this paper.

The concept of the FC system reorganization and negative perturbation feedback is approved for explanation of direct dependence between the characteristic inclination and dimensionless wave velocity. It also explains experimentally observed stability of stationary and quasi-stationary waves ($u \approx 0$). At the same time, usage of this concept for interpretation of experimental data assumes that primary instability is defined by locally minimal value of hydrodynamic clog $H_{\min} < H_{\mathrm{cr}}$.

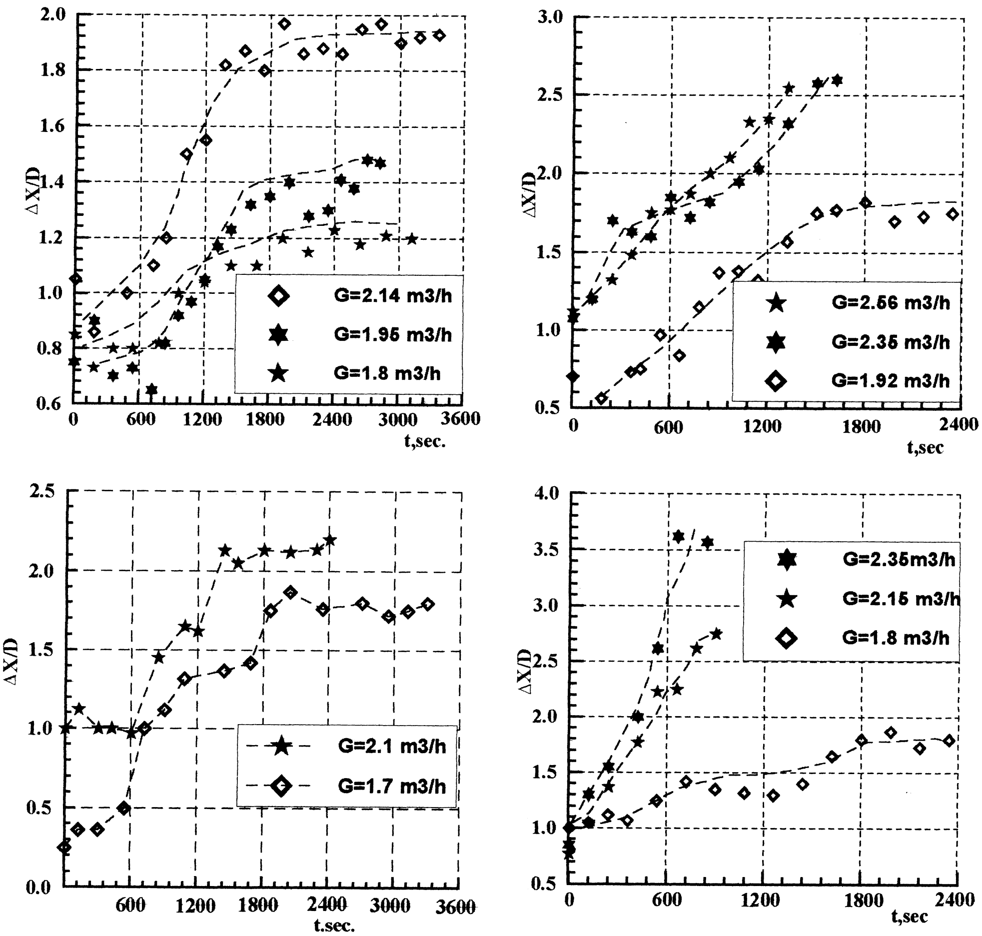

Fig. 4. Dynamics of methane–air filtration combustion inclination. Fuel to air ratio F/A = 1/14, 1/15, 1/16 and 1/17 (equivalence ratio $\varphi = 0.68$, 0.63, 0.595 and 0.56) left to right and up to down correspondingly.

Table 3
Inclination amplitude growth velocity as measured experimentally and estimated by the data of [4]

N series	F/A ratio	u_g, m/s	$u_w \times 10^4$, m/s	$\Delta\dot{X} \times 10^4$, m/s	$\Delta\dot{X}/u_w$	$\langle\Delta\dot{X}/u_w\rangle$
1	1/14	0.99	0.65	0.19	0.29	0.35
	1/14	1.10	0.78	0.26	0.33	
	1/14	1.18	0.9	0.37	0.41	
	1/14	1.24	1.0	0.	0.4	
2	1/15	1.06	0.9	0.41	0.45	0.32
	1/15	1.29	1.6	0.41	0.25	
	1/15	1.38	1.7	0.45	0.26	
3	1/16	0.94	1.0	0.33	0.33	0.27
	1/16	1.12	1.4	0.34	0.24	
	1/16	1.17	1.5	0.35	0.23	
4	1/17	0.99	1.3	0.34	0.26	0.55
	1/17	1.19	1.7	1.1	0.65	
	1/17	1.5	2.4	1.8	0.75	
5	1/15	0.97	1.5	0.9	0.6	0.55
	1/15	1.1	1.95	1.3	0.66	
	1/15	1.33	3.3	1.3	0.39	
[4]	0.0235 (C_3H_8)	2	1	0.36	0.36	0.36

It is shown that if primary instability condition is fulfilled, characteristic inclination can be estimated via the system characteristic length scale and wave dimensionless velocity. In conditions of the reported experiments equation $\Delta X_{ch} = L \cdot u$ gives reasonable results. The inclination amplitude growth at the linear stage may be expressed via the wave front velocity $\Delta\dot{X} \approx 0.4u_w$.

Some physical factors potentially influencing the perturbation dynamics of real systems were mentioned but not analyzed in details. These are: front mean temperature decrease at inclination growth; multi-dimensional character of the front and boundary effects. Contribution of this and other factors to the front inclination dynamics may be examined after detailed numerical and experimental study.

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Appendix A. Estimation of characteristic length of the filtration field relaxation

Isothermal filtration field distribution can be described by Eq. (8). If we assume pressure drop in the system and pressure perturbation in front to be small, compared to output pressure, for excess pressure $p = P - P_0$ we obtain the Laplace equation

$$\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}\right) = 0. \tag{A.1}$$

Assume boundary conditions

$$p|_{x=\text{front}} = p_1 + \delta_P \cos(\pi y/D), \quad p|_{x=L} = 0,$$
$$\left.\frac{\partial p}{\partial y}\right|_{y=0} = \left.\frac{\partial p}{\partial y}\right|_{y=D} = 0.$$

The following solution for the problem can be found

$$p = \delta_P \frac{\exp(-\pi x/D) - \exp(-2\pi L/D)\exp(\pi x/D)}{1 - \exp(-2\pi L/D)} \times \cos\left(\frac{\pi y}{D}\right) + p_1(1 - x/L). \tag{A.2}$$

Corresponding velocity field is

$$u_g(x,y) = \frac{k}{\mu}\left[\frac{\pi}{D}\delta_P\frac{\exp(-\pi x/D) - \exp(-2\pi L/D)\exp(\pi x/D)}{1 - \exp(-2\pi L/D)} \times \cos\left(\frac{\pi y}{D}\right) + \frac{p_1}{L}\right]. \tag{A.3}$$

As follows from (A.2), (A.3), pressure and velocity perturbation relaxes as $\sim \exp(-\pi x/D)$ with the increment not dependent on filtration coefficient, gas properties, temperature or perturbation amplitude. Typical relaxation length is some part of system transverse size $\sim D/\pi$. Due to temperature dependence of gas viscosity $\mu \sim \sqrt{T}$ velocity relaxation will be slowed down relative to pressure, however main dependence persists.

If Eq. (8) is not simplified to (A.1), a more intricate formula of velocity perturbation relaxation depending on P_0, p_1, δ_P, x/D, L and, in case of non-isothermal filtration, on the temperature field, can be obtained. However basic functional dependence remains $\sim \exp(-\pi x/D)$.

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